Government of **Western Australia**
School Curriculum and Standards Authority

WACE
Western Australian Certificate of Education

Western Australian Certificate of Education Examination, 2012

Question/Answer Booklet

MATHEMATICS 3A/3B

Section Two: Calculator-assumed

Please place your student identification label in this box

Student Number: In figures

In words

Number of additional answer booklets used (if applicable):

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction tape/fluid, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Ref: 12-091

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of total exam
Section One: Calculator-free	7	7	50	50	33⅓
Section Two: Calculator-assumed	12	12	100	100	66⅔
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2012*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil**, except in diagrams.

5. The Formula Sheet is **not** handed in with your Question/Answer Booklet.

See next page

Section Two: Calculator-assumed **(100 Marks)**

This section has **twelve (12)** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

Suggested working time: 100 minutes.

Question 8 **(7 marks)**

A ball is dropped from a height of 60 cm onto a horizontal surface. The height reached by the ball after each bounce is two-thirds of the height of the previous bounce.

(a) Write a recursive rule to show the distance travelled by each successive downward motion of the ball. (2 marks)

(b) What is the maximum height reached by the ball after the third bounce, correct to **two (2)** decimal places? (2 marks)

(c) What is the total distance travelled by the ball just as it hits the ground for the seventh time? (3 marks)

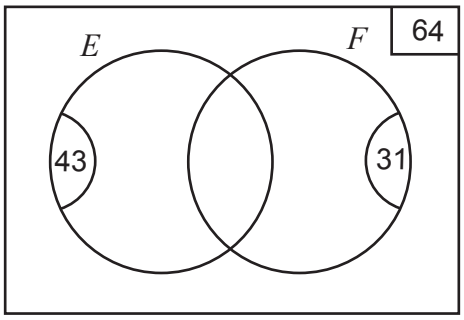
Question 9 **(5 marks)**

E and F are two sets for which $n(E) = 43$, $n(F) = 31$ and $n(U) = 64$.

Using the Venn diagrams or otherwise, determine

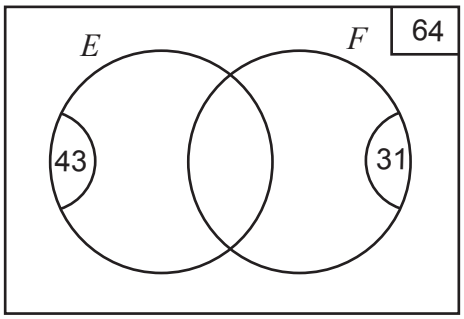
(a) $n(E \cap F)$ if $n(\overline{E \cup F}) = 0$.

(2 marks)



(b) $P(E \mid \overline{F})$ if $n(\overline{E \cup F}) = 10$.

(3 marks)



Question 10 **(12 marks)**

Researchers believe that a lake is inhabited by a native species of fish thought to be endangered. At a point approximately in the middle of the lake, 50 of these fish were captured, tagged and released. The following day, at the same point, another 50 fish were caught and, before they were released, it was noted that 10 of these were tagged.

(a) Show how the researchers could use this information to estimate that the total population (P) of these fish in the lake was 250. (2 marks)

In general, the number of tagged fish caught (t) allowed the researchers to estimate P according to the relationship in this table.

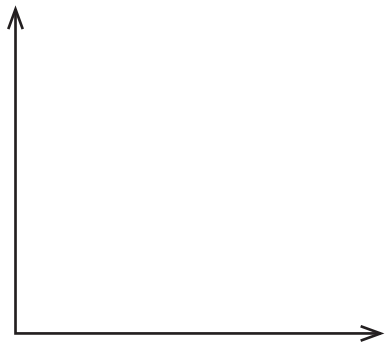
t	5	10	20	25	50
P		250		100	

(b) (i) Complete the table above for population estimates (P) for different numbers of tagged fish caught (t) in lots of 50. (3 marks)

(ii) Describe the relationship between t and P. (1 mark)

(iii) Write an equation for P in terms of t. (2 marks)

(iv) On the axes provided, sketch the relationship found in Part (b) (iii). (2 marks)

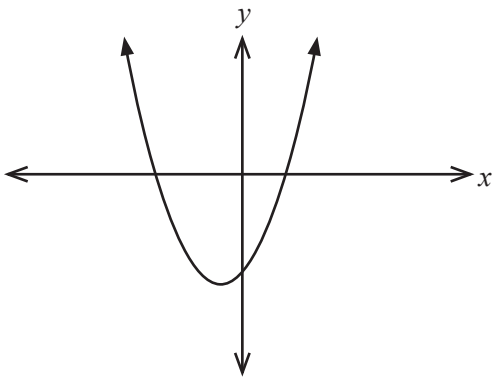


(v) Comment on the rate of change of P as t nears zero and how this might have consequences on the stability of this model for making predictions with very small values of t. (2 marks)

See next page

Question 11 **(10 marks)**

The quadratic function $y = f(x)$ is shown below. The turning point has coordinates (a, b) and y-intercept has coordinates $(0, c)$.



(a) Determine the equation of the line of symmetry for $y = f(x - 1)$. (2 marks)

(b) Determine the coordinates of the turning point of $y = f(x + 4) + 5$. (2 marks)

(c) Determine the coordinates of the y-intercept of $y = -f(x) - 2$. (3 marks)

Another quadratic function $y = g(x)$ has a turning point at $(1, 8)$ and intersects the x-axis at $(3, 0)$.

(d) Determine the coordinates of the x-intercepts of $y = g(2x)$. (3 marks)

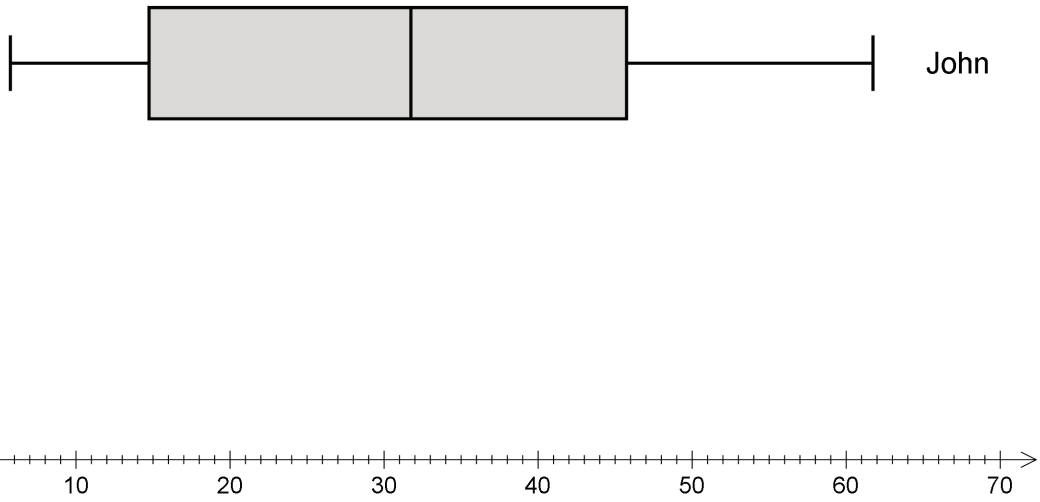
See next page

Question 12 **(7 marks)**

John and Jane work at different phone shops. They recorded the number of phones they sold each month over a period of twelve months. Jane's sales for the twelve months were:

34 47 1 15 57 24 20 11 19 50 28 37

John's data are displayed on the following box and whisker plot.



(a) Calculate the median and interquartile range of phone sales for Jane. (2 marks)

(b) Construct a box and whisker plot for Jane's sales on the above diagram.
(Draw your plot directly underneath John's plot.) (3 marks)

(c) Who had the better sales performance? Justify your answer. (2 marks)

See next page

Question 13 **(3 marks)**

Joanne, a Year 10 student, achieved a final Semester Two mark of 79% in mathematics. The table below gives the weightings and her mean score for the given categories.

	Weighting (%)	Mean score (%)
Homework	10	78
Tests	25	x
Investigations	20	75
Examinations	45	80

Calculate her mean score for tests. Give your answer to the nearest whole percentage.

Question 14 **(4 marks)**

The marks for a mathematics examination at a school are normally distributed with a mean of 54% and a standard deviation of 16.5%.

(a) State the median examination score. (1 mark)

(b) Determine the interquartile range of the examination scores. (3 marks)

See next page

Question 15 **(13 marks)**

A theatre company performed for three weeks at a large venue capable of seating 4200 people. The attendances, in hundreds, at the evening performances, are shown in the following table.

Day		Time (t)	Attendance (in hundreds)	Three-point moving average	Residual
First Week	Wednesday	1	20		
	Friday	2	16	20	−4
	Saturday	3	24	22.7	1.3
Second Week	Wednesday	4	28	26	2
	Friday	5	26	29.3	−3.3
	Saturday	6	34	30.7	3.3
Third Week	Wednesday	7	32	31.7	0.3
	Friday	8	A	34	B
	Saturday	9	41		

(a) Calculate the value of the missing entries marked by A and B. (3 marks)

(b) Calculate the seasonal component for the Wednesday performances. (2 marks)

(c) A regression line was fitted to the three-point moving averages given above. Determine the equation of this regression line correct to **two (2)** decimal places. (2 marks)

(d) This equation was used to represent the trend of the time series. Use this equation, together with the seasonal component for Wednesday, to predict the attendance for Wednesday in the fourth week to the nearest hundred. (4 marks)

(e) Should performances be extended for a fourth week? Give a reason for your answer. (2 marks)

See next page

Question 16 **(12 marks)**

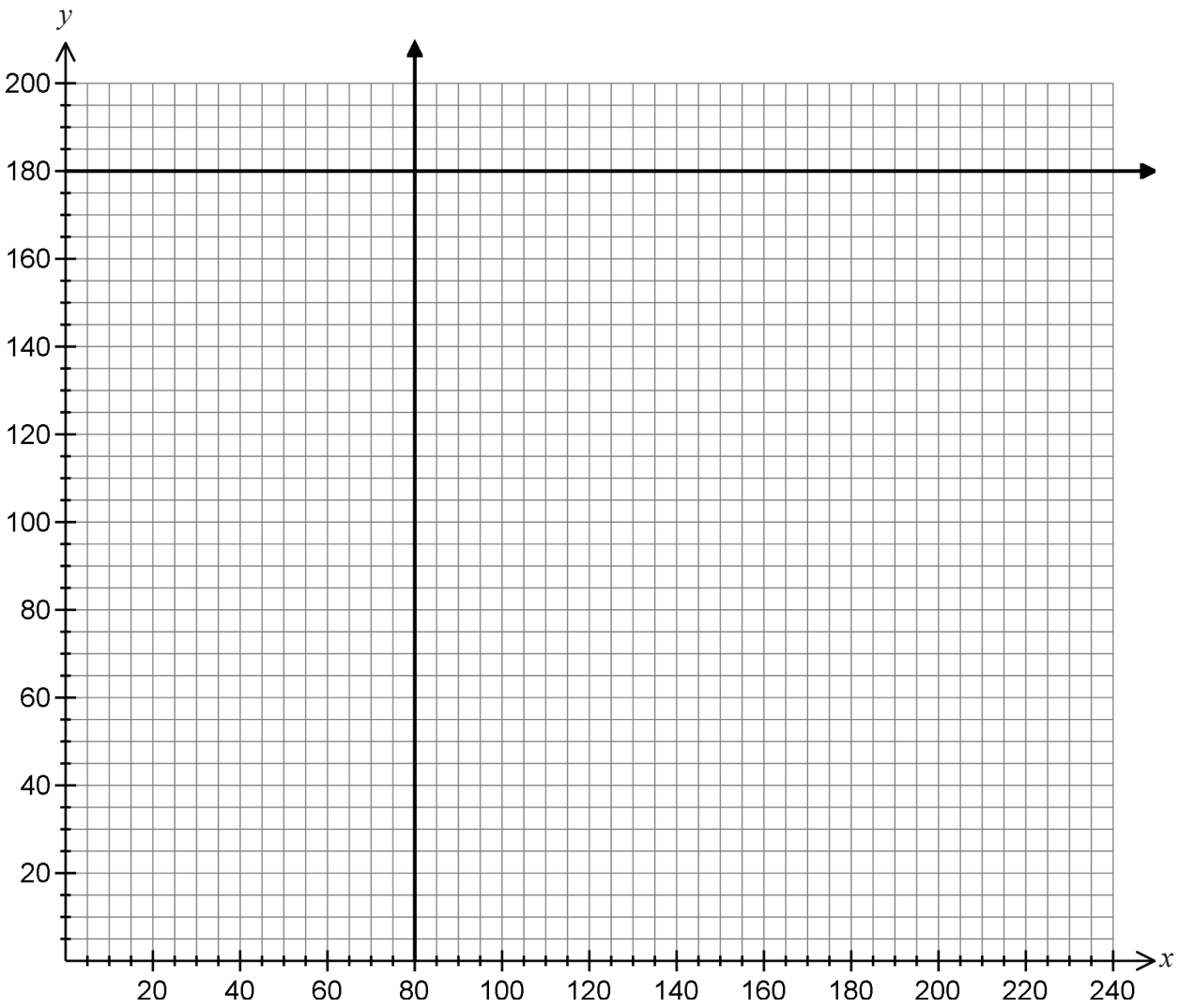
A laboratory that distils natural oils from plant materials produces two types of oil concentrate: an olive oil product for general lubricating use and a refined eucalyptus oil product for use as a cosmetic base and in natural medicines.

Fixed demands require the production of at least 80 litres of the olive oil and at least 70 litres of eucalyptus oil each week. The laboratory has the capacity to produce up to 300 litres of distilled oil each week, of which a maximum of 200 litres can be olive oil or a maximum of 180 litres can be eucalyptus oil.

Let x be the number of litres of olive oil produced and y be the number of litres of eucalyptus oil produced.

(a) Given that $x \geq 80$ and $y \leq 180$, state **three (3)** further inequalities involving x and y, other than $x \geq 0$ and $y \geq 0$. (3 marks)

(b) Graph these inequalities on the grid below and shade the feasible region. (4 marks)



See next page

(c) If the profit on olive oil is $10 per litre and the profit on eucalyptus oil is $20 per litre, calculate the maximum profit the distillery can make each week. (3 marks)

(d) Specific orders require the distillery to produce exactly 160 litres of olive oil in one week. What is the maximum profit it can make in that week? (2 marks)

See next page

Question 17 **(12 marks)**

Simone and Lucy are driving along a highway at constant speeds of 95 km/h and 85 km/h respectively. Lucy drives 15 kilometres further than Simone.

Let the distance Lucy travels be x km.

(a) Write an expression in terms of x for the time taken, in hours, for both Simone and Lucy to complete their respective journeys. (2 marks)

Simone completes her journey in 15 minutes less than Lucy.

(b) Explain why $\frac{x}{85} - \frac{x-15}{95} = \frac{1}{4}$ (2 marks)

See next page

(c) Calculate x and hence state the times taken by Simone and Lucy to complete their respective journeys. Give your answers correct to the nearest minute. (4 marks)

(d) Assuming Simone maintains her constant speed of 95 km/h, at what speed would Lucy need to be travelling to finish her journey in the same time as Simone? Give this speed in metres per second. (4 marks)

See next page

Question 18 **(9 marks)**

Consider the function $f(x) = x^4 - 8x^3 - 270x^2$.

(a) Using calculus techniques, show that the function has two stationary points for the domain $-2 \leq x \leq 20$. (3 marks)

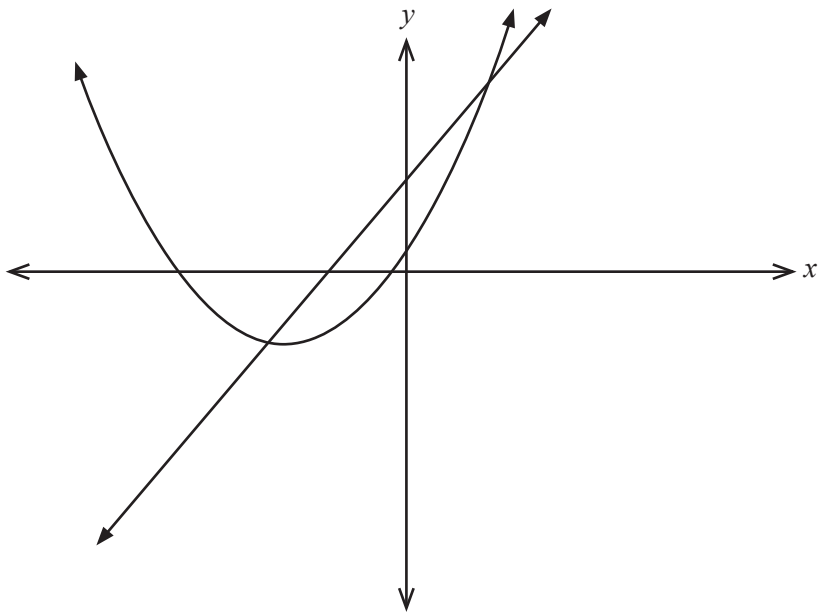
(b) Determine the coordinates of the stationary points identified in Part (a) and state their nature. (3 marks)

(c) For the domain $-k \leq x \leq k$, where k is an integer, the function has a global maximum of 0 and a global minimum of $-25\,000$. Determine the value of k. (3 marks)

See next page

Question 19 **(6 marks)**

The functions $f(x) = x^2 + 4x + 1$ and $g(x) = 3x + d$, where d is a constant, are shown below.



For what value(s) of d does the equation $f(x) = g(x)$ produce:

(a) one solution? (5 marks)

(b) two solutions? (1 mark)

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

Additional working space

Question number: _____

Additional working space

Question number: _____

Published by the School Curriculum and Standards Authority of Western Australia
27 Walters Drive
OSBORNE PARK WA 6017